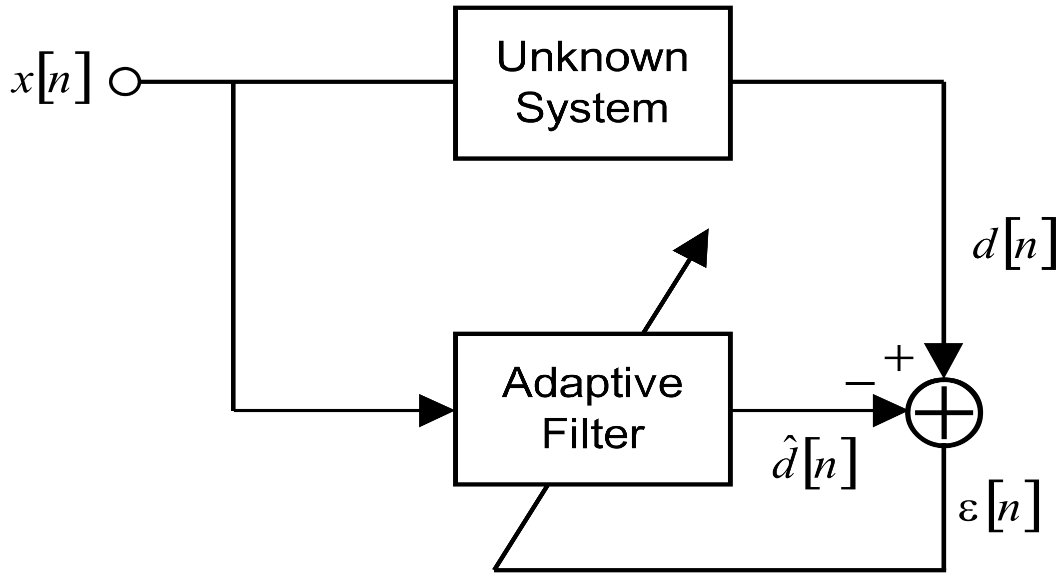
Selected examples for Chapter 11.

EXAMPLE 11.1

An illustration of the LMS method for system identification is considered here.



In this experiment the unknown system is described by

$$d[n] = x[n] + 0.7x[n-1]$$

A first order adaptive filter is used ($P = 2$), so that the weights w_0 and w_1 should converge to 1.0 and 0.7 respectively.

The input sequence $x[n]$ is chosen to be a first order process of the form

$$x[n] = ax[n-1] + w[n]$$

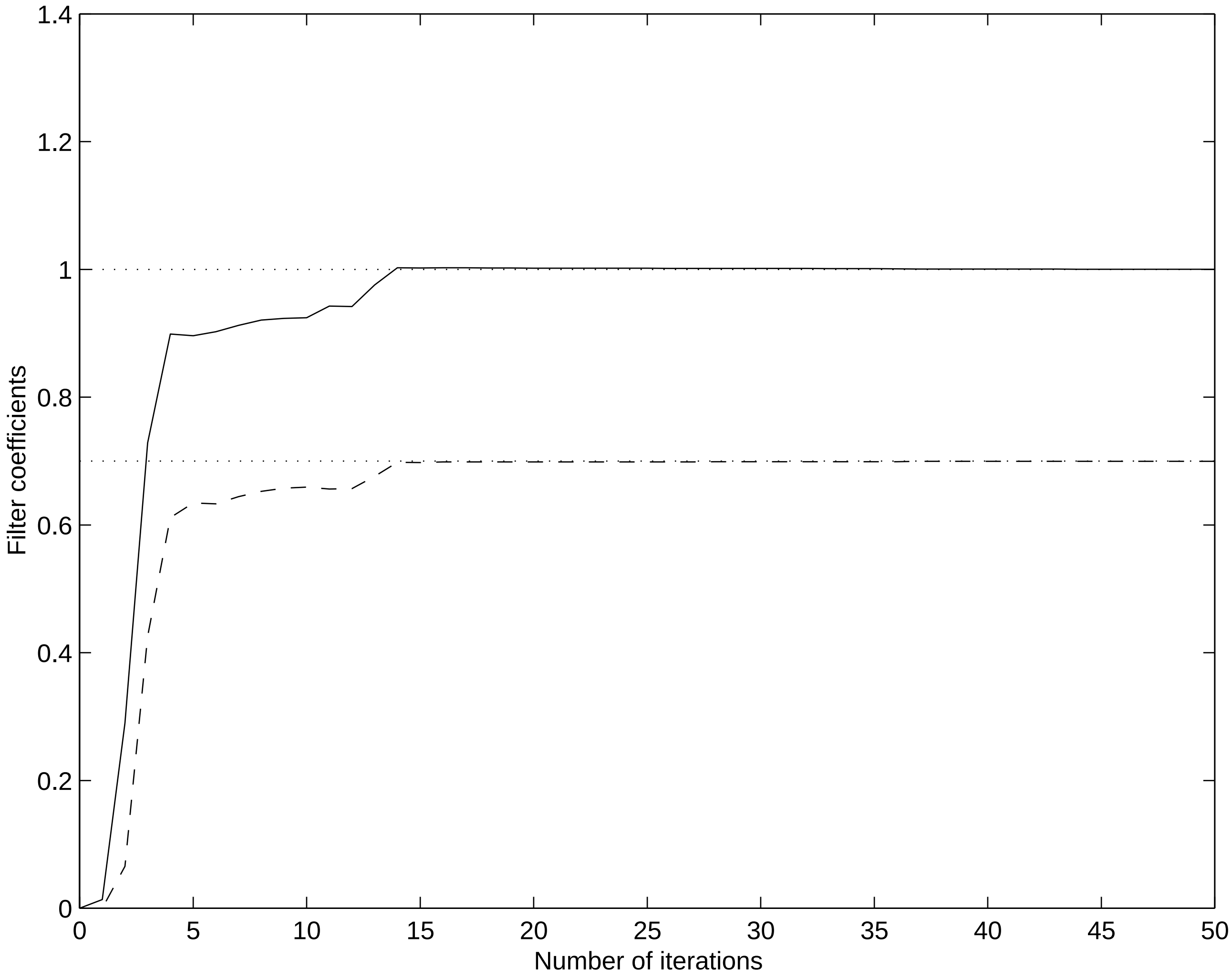
where $w[n]$ is white noise with variance $\sigma_w^2 = 1$. The parameter a determines the eigenvalue spread of the input process (see Problems). Some values of a, the corresponding eigenvalue spread χ, and the upper bound $(2/\lambda_{\max})$ on μ are given in the table below.

a	χ	*upper bound*
0	1	2
0.25	1.667 (1.59)	1.5 (1.54)
0.5	3 (2.8)	1 (1.04)
0.75	7 (6.3)	0.5 (0.54)
0.95	39 (31)	0.1 (0.12)

The main numbers listed are the theoretical values. The experimental values estimated from the data are shown in parentheses for comparison. The first case listed $a = 0$ corresponds to pure white noise and is the ideal case for rapid convergence.

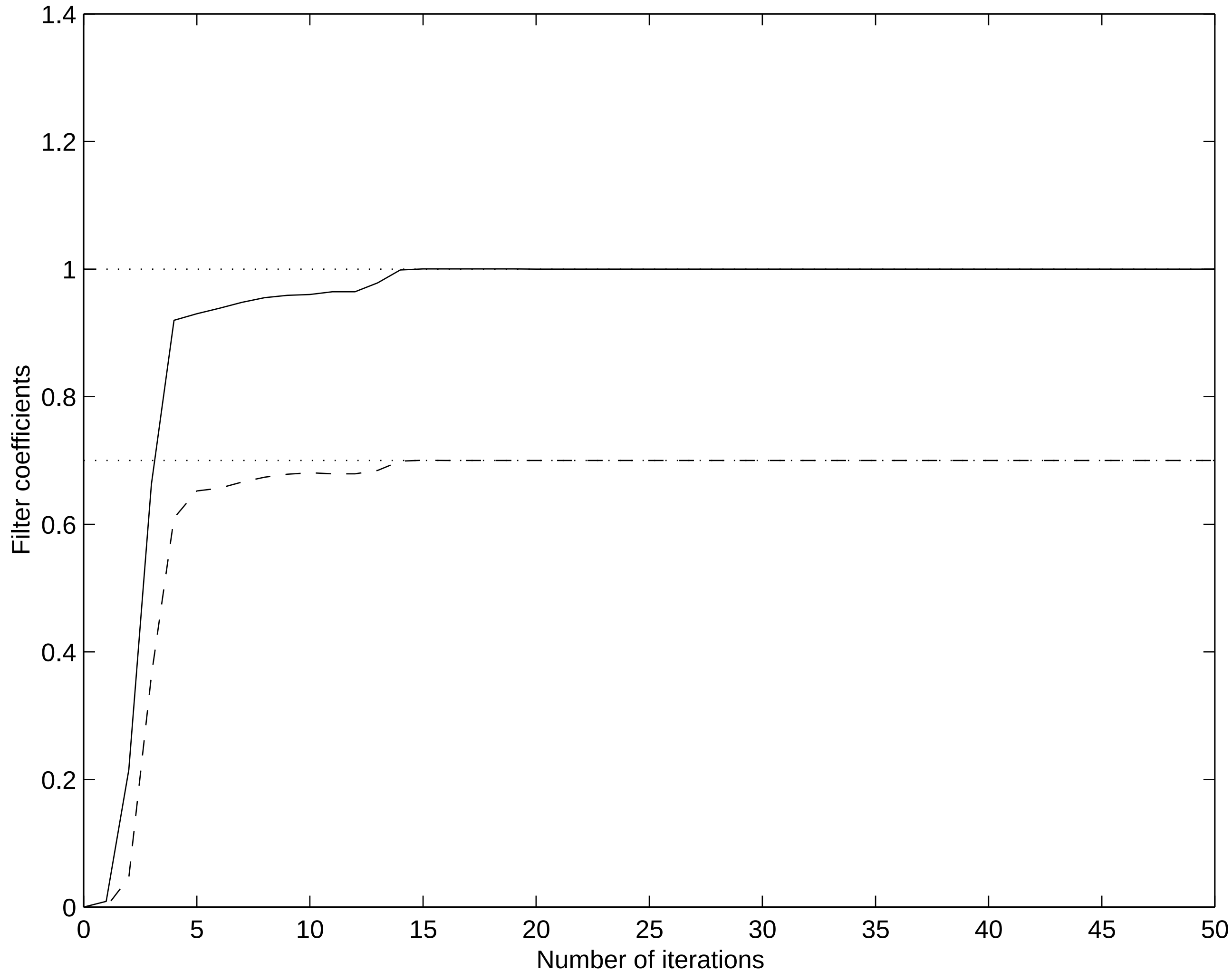
The set of four figures shows the trajectories of the filter coefficients for different values of a. In each case, μ was taken to be $\frac{1}{10}$ of the upper bound for that case. Notice that the settling time increases as a and thus the eigenvalue spread increases. In the first two cases the filter coefficients have converged to the correct values (shown by the dotted lines) within less than 15 iterations. In the third case ($\chi = 7$) about 100 iterations are required for convergence, while in the last case the coefficients still have not reached their final values in 500 iterations.

Weight trajectories for $a = 0.25$ $(\chi = 1.667)$ $\mu = 0.15$



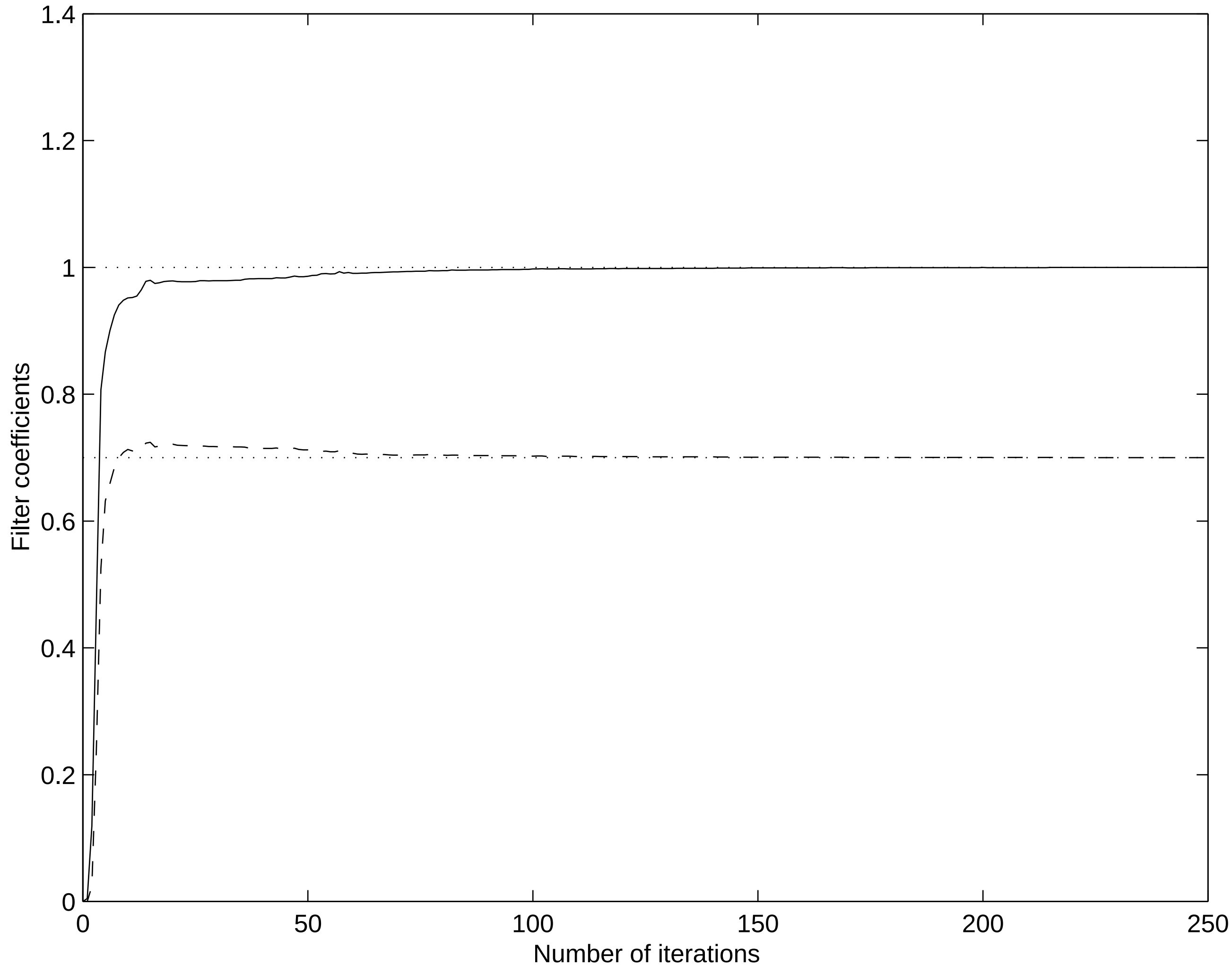
Ex.11.1(4)

Weight trajectories for $a = 0.5$ $(\chi = 3)$ $\mu = 0.1$



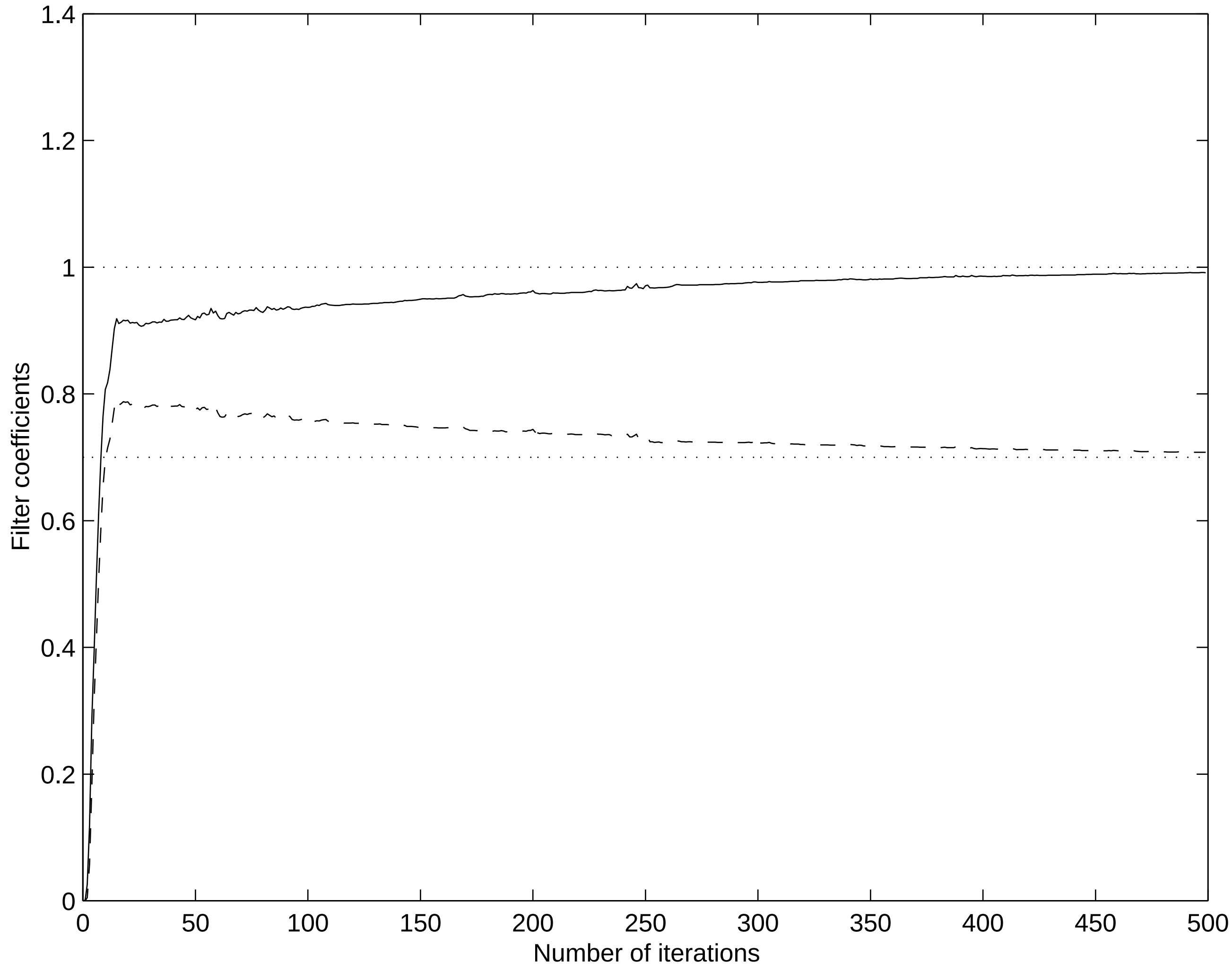
Ex.11.1(5)

Weight trajectories for $a = 0.75$ $(\chi = 7)$ $\mu = 0.05$



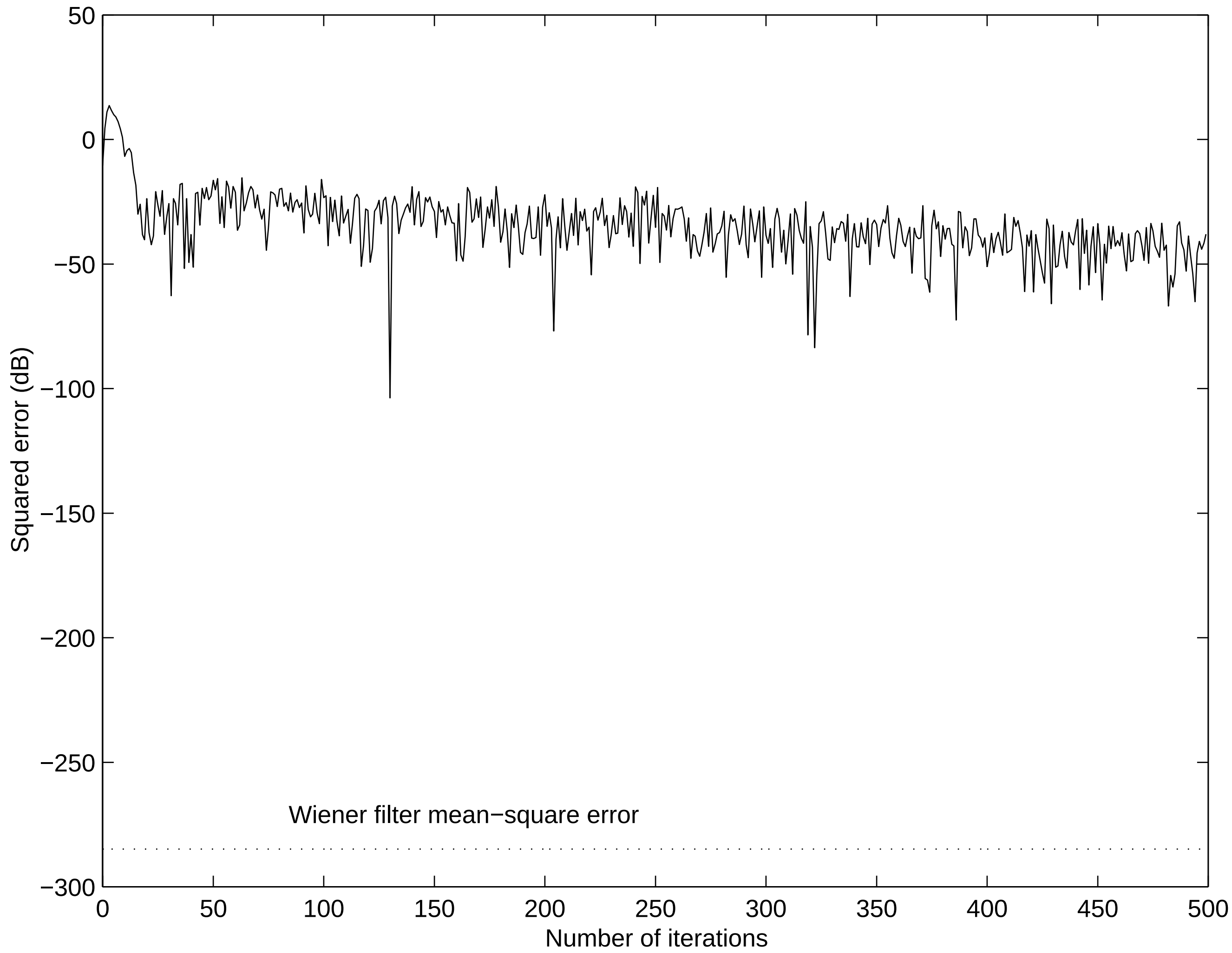
Ex.11.1(6)

Weight trajectories for $a = 0.95$ $(\chi = 39)$ $\mu = 0.01$



Ex.11.1(7)

The experimental learning curve (i.e., squared error) for this last case is depicted below:



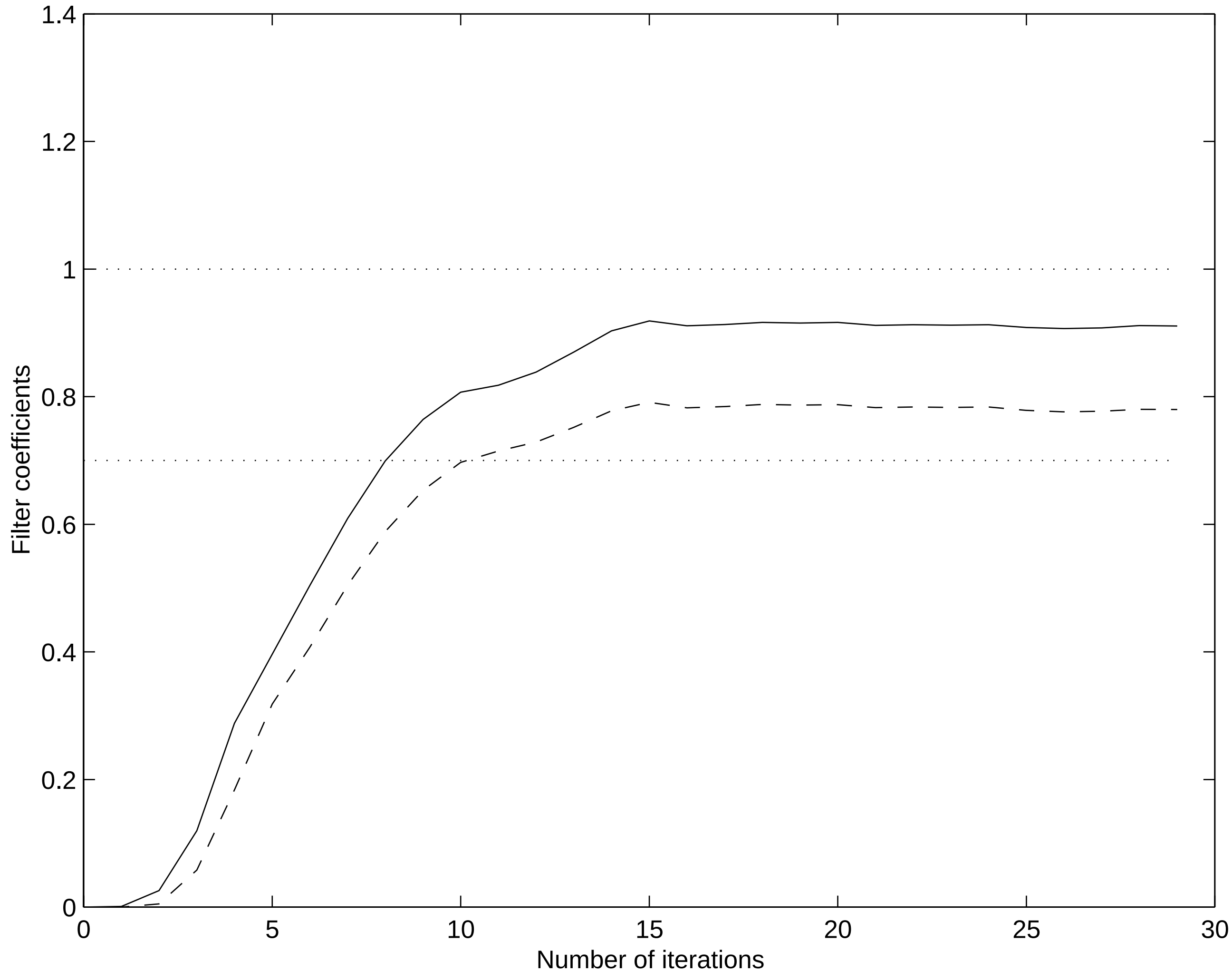
□

Ex.11.1(8)

EXAMPLE 11.2

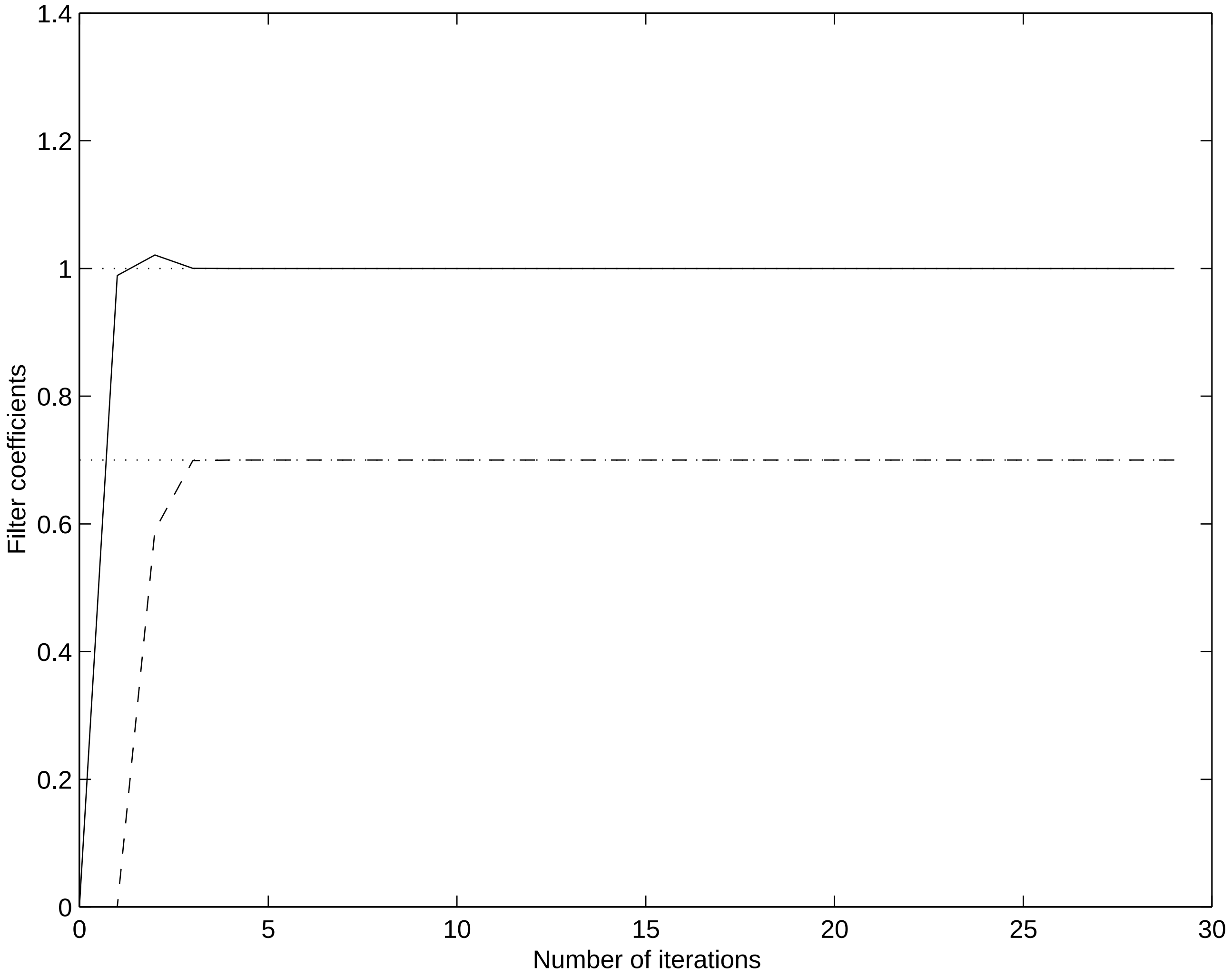
The RLS method is applied to the system identification problem considered in the previous example. Recall that when the input process is highly correlated convergence is very slow. The plots in this example compare the performance of LMS and RLS during the first 30 iterations. From this limited observation time it appears that the values of the filter coefficients produced by the LMS algorithm are leveling off but are nowhere near the true values. (Actually we know from the previous example that the filter coefficients have *not* leveled off and do approach the true values, but only after more than 500 iterations.) The RLS algorithm however converges to the correct values after just *three* iterations and remains stable. This remarkably better performance is obtained, of course, with a significant increase in the number of computations per iteration.

Weight trajectories for LMS method $a = 0.95$ $(\chi = 39)$ $\mu = 0.01$



Ex.11.2(2)

Weight trajectories for RLS method with same input



$\square$

Ex.11.2(3)